TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No.20.

NOTES ON SPECIFICATIONS FOR FRENCH AIRPLANE COMPETITIONS.

By

W. Margoulis.

Translated from the French by

Paris Office, N.A.C.A.

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The following translation from "L'Auto," of the rules officially adopted by the Aeronautical Sporting Commission of the Aero Club of France, at its meeting on February 6th, was made by the Paris Office of the National Advisory Committee for Aeronautics, in view of the general interest that has been evidenced in the Gordon-Bennett and other aeronautical competitions.

The Competition of 1920.

The following Rules were officially adopted by the Aeronautical Sporting Commission of the Aero Club of France at its meeting on February 6th.

The newspaper "L'Auto" offers a Prize of 10,000 francs for a test flight, to be known as the "Prize for Greatest Speed Range." The test flight can be made at any time between April 1st and July 1st inclusive. The Prize will be awarded to the pilot who succeeds in obtaining the highest maximum and lowest minimum speeds, and in landing within the shortest distance. One classification will be made of the speed ranges and another of the landing distances, the final award depending on the result of the two lists. In case of a tie the competitors will be placed according to the Speed Range classification.

The Test Flight must be made in France on the Villacoublay aerodrome, where a base of 3 kilometers will be established by the organizers

The Competitors must make the Test Flight alone in the machine; the flight can be made only in a wind of less than 10 meters per second, the speed being measured at 10 meters from the ground by the Commissioners. The flight must be accomplished by the aid of usual flying tactics only, all stunt flying, such as looping, being excluded.

The Competition is limited to machines of Class C, (engine driven airplanes).

The Entry Fee is 500 francs, which will not be returned. The Entry Fees will be used to provide additional prizes, viz.: one-half to the Competitors placed second; one-quarter to those classed third; one-eighth to those classed fourth; one-sixteenth to those placed in the fifth class, etc., etc., at the same decreasing rate for all classes of competitors.

The Prize for Greatest Speed Range is only open to French Pilots flying machines built in France by a French Constructor or a French Society.

The Entry Form must be signed by the pilot. Entry Forms may be sent to the offices of "L'Auto" 10, rue du Faubourg Montmartre, on and after March 1st, 1920. Applications for entry must be made in writing on the Entry Form adopted by "L'Auto", and accompanied by the sum of 500 francs, eight days before the date of the first test. The Test Flights will commence at Villacoublay on April 1st at 8 a.m. They will take place twice a week, on Tuesdays and Thursdays. Entrants must inform the organizers of the day and hour of their test flight before the Saturday of the week preceding the time chosen.

The Competition will be conducted as follows:

1st. Test of Maximum Speed. A straight line of 3 kilometers will be marked off by pylones on the Villacoublay aerodrome. The entrants must cover this distance - there and back - at maximum speed. The mean time taken for the double journey will then be calculated by the times (going and returning) noted, allowing for the velocity of the wind measured by anemometer at the time of the test flight.

The flight should be practically horizontal.

The interval between the end of the first part of the flight (going) and the beginning of the return journey must be less than five minutes without landing. During that interval the competitor should keep outside the track.

2nd. Minimum Speed Test under the same conditions, the measurements will again be taken for the same machine flying horizontally at its minimum speed.

If <u>t</u> is the average time taken to cover the distance marked at maximum speed, and <u>T</u> the time required to cover the same distance at minimum speed, the machines will be placed according to the values of the ratio T/t.

Each machine entered shall have the right to make three attempts, not necessarily on the same day.

The machines must undergo no modification for the two tests - the maximum and minimum speed - and must carry the same useful weight for each test. In changing the speed of flight, the pilot must use only the controls of the engine or of the machine. Wing manipulation can only take place during flight.

The tests will be made at a practically constant height of 500 meters; the competitors must have on board a barometer registering up to a maximum

of 1000 meters, sealed by the commissioners. This barometer must be accepted by the commissioners. The differences of altitude must not exceed 50 meters more or less than the prescribed altitude. The machine must not leave the given track of 100 meters in width.

The tests will be made individually; the commissioners will, if necessary, fix the order in which entrants will fly, by the drawing of lots.

Entries are valid during the whole of the time the competition continues, and allow the entrants to make the three attempts as stated in Rule 7.

A fall puts the competitor out of the running.

The shortest distance Landing Test will be made under the following conditions:

A sanded area, finely raked, measuring 200 meters along one side, will be made at a point on the given track. On this area the competitors must land.

The distance travelled on the ground at landing will be measured as follows: A line will be traced perpendicular to the rear plumb line of the traces of the wheels of the airplane at its first contact with the ground; then will be taken the perpendicular passing through the axis of the wheels when the machine has stopped, and this line will be projected on the ground.

The distance between the two perpendiculars shall be the distance covered at landing.

The time-keepers' expenses shall be borne by the entrants.

The newspaper organizing this Competition can in no case be held responsible for any accident occurring during the Competition, either to pilots or others.

ENTRIES

Entries will be received at the offices of "L'Auto" from March 1st. Would-be Competitors must send their request to the "Auto"; an Entry Form will then be supplied, which they must return, duly filled in and signed, together with the Entry Fee of 500 francs.

The list of Entries will be closed on March 31st at 6.30 p.m.

General Principles.

The five years of War have brought about a remarkable development of aviation in all its different departments. Let us hasten to add that we cannot have a very clear idea of all the progress accomplished until after our return to normal peace conditions, the manufacturers of the two enemy camps having naturally hidden their designs from each other. Within the very near future, the rivals on both sides will learn what their recent enemies have been doing.

Meanwhile, whatever may be the general opinion, there is no problem more urgent than that of security. The complete statistics of accidents occurring quite recently, give the following results (excluding, of course, war accidents).

Falls when the machine is near the ground, 86%.

Falls in full flight: 14%.

The latter item, moreover, diminishes year by year. The 86% of the first class of accident is divided as follows:

Starting accidents (getting off) 19%.
Accidents at landing: 67%.

And this latter class does not seem to diminish at all, on the contrary.

Also, the steadily increasing speed of flight renders landing a more and more ticklish thing to effect, especially on a strange ground. The obvious conclusion is that we must encourage all investigations having for their object the minimizing of landing difficulties, and of the distance to be traversed before stopping.

The new Michelin prize is the outcome of this movement, and the solution of the problem is hoped for in the helicopter type machine.

Considering only the airplane - the current solution at present - we can understand the advantage of flying at high speed; but, at the same time, a very low landing speed is necessary for security's sake. From these two requirements has arisen the Speed Ratio Competition, the Rules of which have just been given.

These introductory remarks were necessary, but we must not forget that the idea of such a test dates back ten years, and that its author was the Marquis de Dion, who thus gave another proof of his practical common sense. From the time of the first landing accidents - and Heaven knows how painful they were - M. de Dion was impressed with the necessity of reducing landing speeds. This Competition should, therefore, be placed under his patronage. There is no greater authority in the matter than he.

C. FAROUX.

CONCERNING THE RULES OF THE "HIGHEST SPEED RATIO" COMPETITION."

The ratio of the maximum speed in horizontal flight at the altitude of practical utilization of the airplane, to the landing speed, has been called by me* SPEED RATIO.

Assuming horizontal flight at 2000 m. and the ceiling at 4000 m., which corresponds to present day transport planes, we find that the Speed Ratio for such planes is 1.88, that is, if we take a minimum landing speed of 90 km/h, the maximum speed of the transport airplane at 2000 m. will be 90 x 1.88 = 169 km/h. I then showed that this Speed Ratio, WHICH LIMITS TRANSPORT PLANES TO COMPARATIVELY LOW SPEEDS, may be increased:

1st. By increasing the altitude of the ceiling, while keeping the altitude of flight under the ceiling, which would allow of the ratio being increased proportionally to

$$\sqrt{\delta_{2000}/\delta_{H-2000}},\quad \delta_{2000} \text{ and } \delta_{H-2000}$$

being respectively the specific weights of the air at the altitude of 2000 m. and at 2000 m. below the ceiling. Thus, for flight at 8000 m. with a ceiling of 10000 m., the speed ratio would be 1.88 x 1.41 = 2.65; we should thus obtain a speed of 90 x 2.65 = 238 km/h.

But the useful load would be diminished on account of the increase of engine power; this increase is proportional to $(\delta_{2000}/\delta_{H-2000})^{3/2}$. T_{2000}/T_{H-2000}, T_{2000} and T_{H-2000} being respectively the absolute temperatures of the air at 2000 m. and at 2000 m. below the ceiling. In the above example

$$(\delta_{2000}/\delta_{8000})^{3/2} \times T_{2000}/T_{8000} = 3.06.$$

The power of the engine on the ground and therefore its weight must thus be tripled; but the consumption of fuel required for covering a given distance remains the same as in flight at 2000 m., assuming, of course, a zero wind and an invariable specific consumption of the engine.

2nd. By increasing both the ceiling and the distance separating the altitude of practical flight from that of the ceiling. In this case, however, we should not only have an increase in the power, and consequently, in the weight of the engine, but also an increased weight of fuel to carry in order to cover a given distance.

The increase of speed ratio due to flight with smaller lift has a limit corresponding to flight at minimum drag; the limit value of the ratio for the 1918 airplanes is 2.04 instead of 1.88, that is, only 8% greater than in the first case. If, as in the preceding example, the machine has a ceiling of 10000 m., but is flying near the ground instead

*See the last publication of the works of the Eiffel Laboratory "Resumé of the principal works executed at the Eiffel Laboratory during the War," p.2.

of at 8000 m. the ratio will be (without allowing for variation of temperature) 2.04 x 1.41 = 2.88 and the speed near the ground 90 x 2.88 = 259 km/h; the consumption of fuel for a given distance is 0.355/0.130 = 2.73 greater than that of the machine flying at 8000 m., 0.355 and 0.130 being the respective values of R_X/R_Y at 2000 m. and at 10000 m. below the ceiling.

3rd. Finally, the same increase of speed as in the first case can be realized by the use of a supercharged engine which would still allow of flying at 2000 m. below the ceiling, but with a much higher ceiling, the power of the engine on the ground being increased only in the ratio of

$$\sqrt{\sigma_o/\ \sigma_{(H-2000)}} \times \left[(\ \delta_{2000}/\ \delta_o)^{3/2} \times T_{2000}/T_o\right]$$

that is, 1.1 for flight at 8000 m. We must also allow for the increase of weight due to the supercharging device.

Also, while possessing sufficient excess power, the supercharged engine would allow of flying at an incidence nearer the optimum incidence (R_X/R_Y minimum); more especially, we can fly at an incidence corresponding to that of an ordinary airplane flying at 2000/3, that is, 670 m. below the ceiling (since the supercharged engine triples the ceiling of an airplane fitted with an ordinary engine). Under these conditions, the speed ratio for flight at 8000 m. would be only 2.16 instead of 2.88, but on the other hand, the engine power and consumption (the value of R_X/R_Y being lower) would be diminished, so that we should have to look further into the question in order to know whether it would be advantageous to increase the incidence of flight.

We will give an example enabling us to take into account the influence of the increase of power on the useful load of the machine in the two cases: ordinary engine and supercharged engine. A good present day airplane which lands at 90 km/h and has its ceiling at 4000 m., carries 40 kg/sq.m and weighs 6.8 kg/HP. We will assume that the weight of the engine set is 1.5 kg/HP and that the weight of the glider is 30% of the total weight of the airplane. The following Table gives the numerical characteristics for the three cases.

	Flight at 2000 m.	Flight at 8000 m. with ordinary engine	Flight at 8000 m. with supercharged engine.
Speed Ratio	1.88	2.65	2.65
Max. Speed in km/h	169	238	238
Weight per HP on ground	6.8	6.8/306 = 2.23	6.8/11 = 6.2
Weight of engine set in % of total weight of plane.	1.5/6.8 = 22%	1.5/2.23 = 67%	1.5/6.2 = 24%
Weight of glider / Tot. Weight Plane	30%	30%	30%
Useful Load	48%	3%	46%

By this we see that in the second case the machine could not carry a useful load, while the adoption of a supercharged engine would allow the speed to be increased from 169 to 238 km/h without any appreciable reduction of useful load*.

It is easy to show that for airplanes having the same reduced polar(1) the Speed Ratio depends solely on the ceiling and on the altitude of utilization.

If we compare the Standard REDUCED POLARS (2) of airplanes turned out from 1913 to 1919 we note that the speed ratios (maximum speed at 2000 m., minimum speed near the ground, ceiling 4000 m.) corresponding to these polars, have increased from 1.33 to 1.88, that is, by 41%.

We will indicate the formulas giving the values of the Speed Ratios as function of the aerodynamic and mechanical characteristic coefficients of the airplane, and of the characteristics of the atmosphere at the altitude of the ceiling and of flight at maximum speed.

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* See Note 1, p. 11.

(1) For comparing airplanes from an aerodynamical point of view, I established a new method in 1915 which has proved fruitful in practice. This method consists of reducing the polars of airplanes to the surface of 1 sq.m., that is, of dividing the values of the elements of the resultant by the area of the airplane. The polar thus obtained will be the REDUCED POLAR of the airplane.

(2) Comparing the reduced polars of airplanes constructed during a certain period, we see that most of them differ very little from each other, so that the airplanes of each year, say, may be characterized by a well determined reduced polar. This I have called the STANDARD REDUCED POLAR.

We will call:

$$\Xi = \frac{V}{V_m} = \frac{\text{Maximum speed}}{\text{Minimum speed near the ground}}$$

K_Y^m Maximum lift of airplane.

K_X/K_Y Fineness ratio of airplane at incidence of flight at maximum speed.

$(K_Y^{3/2}/K_X)_H$ The characteristic coefficient of the reduced polar corresponding to flight at the ceiling.

P_m MOTIVE power of the engine on the ground at the number of revolutions corresponding to horizontal flight near the ground at maximum speed.

Q the weight of the airplane.

S the lifting surface.

δ_o and T_o - Specific weight and absolute temperature of the air on the ground.

δ and T - Specific weight and absolute temperature of the air at the altitude of flight at speed V.

δ_H and T_H - Specific weight and absolute temperature of the air at the ceiling.

f(V) - Reduction of the useful power of the engine set (with respect to the useful power on the ground) for flight at speed V.

$f(V_H)$ - The same reduction for flight at ceiling.

ρ_o - Efficiency of propeller in horizontal flight near the ground at maximum speed.

We have:

$$\Xi = \sqrt{\frac{S}{Q} \cdot \frac{P_m}{Q} \times \frac{T}{T_o} \cdot \frac{\delta}{\delta_o} \times \rho_o \cdot f(V) \times \frac{\sqrt{K_Y^m}}{K_X/K_Y}}$$

or

$$\Xi = \frac{\left(\frac{\delta_o}{\delta_H}\right)^{3/2} \cdot \frac{T_o}{T_H}}{\frac{\delta_o}{\delta} \cdot \frac{T_o}{T}} \times \frac{f(V)}{f(V_H)} \times \frac{\sqrt{K_Y^m}}{\frac{K_X}{K_Y} \cdot \left(\frac{K_Y^{3/2}}{K_X}\right)_H}$$

If the maximum speed test is made near the ground, as in the Competition, $T = T_o$, $\delta = \delta_o$ and $f(V) = 1$. Under these conditions, the foregoing formulas are written as follows:

$$E = \sqrt{\frac{S}{Q}} \cdot \frac{P_m}{Q} \cdot \rho_o \frac{\sqrt{K_Y^{\,m}}}{K_X/K_Y}$$

or

$$E = \left(\frac{\delta_o}{\delta_H}\right)^{3/2} \quad \frac{T_o}{T_H} \quad \frac{1}{f(V_H)} \quad \frac{\sqrt{K_Y^{\,m}}}{\frac{K_X}{K_Y} \cdot \left(\frac{K_Y^{3/2}}{K_X}\right)_H}$$

The demonstration of these formulas will be found in NOTE II annexed to the present study, p. 12.

From what precedes it will at once be seen that the airplane winning the Speed Ratio Competition must have the highest ceiling and must therefore also realize the lowest aerodynamical qualities produced by $Q/P_m \times \sqrt{(Q/S)}$. Now, no useful load is imposed by the Regulations and as, on the other hand, the weight of fuel to be carried is very small, we may estimate the total load at 100 kg. corresponding to the weight of the pilot and the fuel carried.

We will try and see which type of airplane among those existing, has the best chance of carrying off the prize.

We will call:

Q - The total weight of an airplane with the complete load (q_{ch}) which it carries in practice.

q_p - The weight of the glider.

q_m - The weight of the eng_ine set.

$\frac{Q}{S}1^*$ and $(Q/P_m)1^*$ - The weight per square meter and per HP of the airplane after being lightened for participation in the Competition.

a - The weight per HP of the engine set.

It is easy to show that:

$$(Q/S)1^* = Q/S \ (q_p/Q + aP_m/Q + 100/Q)$$

$$(Q/P_m)1^* = 1 + q_p/Q \times Q/aP_m + 100/aP_m$$

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* 1 is an index.

We thus see that the values of Q/S and Q/P_m for the lightened airplane, and consequently the product $Q/S \times Q/P_m$ will be all the smaller, as the load per sq.m. and per HP of the original airplane was smaller.

For several actual airplanes which seem to me the most likely winners in the Competition, I give in the following Table the values of the Speed Ratio (ratio of maximum and minimum speeds near the ground) and also the values of the ceiling, determined by means of my ABACUS FOR DETERMINING THE PERFORMANCES OF AN AIRPLANE.

From what has just been said, it will be remarked that the conditions of the Competition are the same as those for a Ceiling Competition, and we note the agreement between the performances determined by the abacus and the actual performances of the airplanes mentioned.

We believe, however, that the minimum speeds will be slightly less than those we indicate, so that the Ratios will, in reality, be rather larger.

We admit, of course, that the controls of the machines have still sufficient power at low speeds to enable the machine to fly at these low speeds near the ground without danger, as required by the Regulations.

Type of Machine	Nieuport* 29 C.1	Curtiss "Wasp"	Breguet* "Berline"	Bristol* "Bullet"
Description	Single-seater Pursuit plane Hispano engine	Single-seater Pursuit plane Curtiss engine	Transport Plane for 7 men Renault engine	Single-seater Pursuit plane Cosmos engine
Kg/sq.m	$\frac{860}{27} = 31.9$	$\frac{960}{29} = 33$	$\frac{1350}{72} = 18.8$	$\frac{870}{27.5} = 31.6$
Kg/HP	$\frac{860}{310} = 2.78$	$\frac{960}{400} = 2.4$	$\frac{1350}{450} = 3$	$\frac{870}{450} = 1.94$
Speed Ratios	$\frac{235}{80} = 2.94$	$\frac{252}{81} = 3.1$	$\frac{195}{63} = 3.1$	$\frac{268}{79} = 3.4$
Ceiling in m.	8800	9500	10000	10600

We thus see that the rules of the Competition favor specialized machines or those which have been specially transformed for the Competition, and that no account is taken either of the aerodynamic qualities of the machines nor of their practical qualities for the work for which they were designed.

* For a fuller description see "The Paris Air Show, 1919" (on file in the Office of Aeronautical Intelligence of the National Advisory Committee for Aeronautics).

For a Speed Ratio Competition to be of service to Aviation, I consider that the rules should provide for two tests. The first would serve to classify ALL airplanes taking part in the Competition by their aerodynamic qualities. For this purpose would be determined for each airplane the speed ratios corresponding to maximum horizontal speed at 2000 m and to minimum speed near the ground, THE AIRPLANE BEING LOADED SO AS TO HAVE ITS CEILING AT 4000 m. For machines not having their ceiling at that altitude, the maximum speed test would take place near the ground and the speed ratios would be corrected as indicated above.

The second test, intended for the comparison of the Speed Ratios of airplanes used for the same kind of work, would consist of a maximum speed test at the altitude of practical utilization of the airplane, and a minimum speed test near the ground.

The different categories in which the airplanes would be classified, and the altitude (H) of the maximum speed tests, might be as follows:

MILITARY AIRPLANES.

I. - Pursuit Planes, single-seaters - H = 6000 m.
II. - " " two-seaters, and Army Corps Planes - H = 5000 m.
III. - Night Bombing Planes - H = 4000 m.

TRANSPORT AIRPLANES.

For all airplanes - H = 2000 m.

SPECIAL AIRPLANES.

No limitation of useful load; maximum speed test to be made near the ground.

REMARK. - The altitudes indicated are minimum; competitors wishing to fly at greater altitudes would be free to do so; such might especially be the case with AIRPLANES FITTED WITH SUPERCHARGED ENGINES.

Military Airplanes would have to carry the regulation useful load and amount of fuel; Transport Planes would carry, say, 100 kg. per passenger and fuel for 600 km. with a contrary wind blowing at 50 km/h.

We think that with rules such as we have just sketched, the Speed Ratio Competition might usefully contribute to the progress of Aviation, while the rules as now laid down lead to the participation in the Competition of special airplanes, or of machines specially transformed for the occasion and which are of no practical use.

NOTE 1. - To be complete this Table should also comprise the case of the airplane with SUPERCOMPRESSED engine and with engine SUPERCOMPRESSED AND LIGHTENED.

We may estimate that for flight at 8000 m., the weight of the set with supercompressed engine would be equal to $0.67 \times 0.94 = 0.63$ of the total weight of the airplane, so that the useful load (7%) would still be

too small. As regards the supercompressed and lightened engine, it has not yet been built for an altitude of 8000 m., but, because of its great weight and low mechanical efficiency, we do not think it would be more advantageous at high altitudes than the supercharged engine.

NOTE 2. - ESTABLISHMENT OF THE FORMULAS FIGURING IN THE PRECEDING TEXT.

For the Notation, see p. 9.

Section 1. - VALUE OF THE SPEED RATIOS IN FUNCTION OF THE ALTITUDE OF FLIGHT AT MAXIMUM SPEED.

In what follows I admit that the relation between the specific weight of the air and the altitude is:

$$h = 21400 \operatorname{Log} (\delta_o / \delta) \qquad (1)$$

which corresponds with sufficient approximation to the Standard Atmosphere of the S.T.Aé. ($T_o = 288^o$, $\rho_o = 760$ mm., decrease in the temperature by Radau's Law.)

The values of the altitude, the characteristics of the Standard Atmosphere, and the values of f(V)* may be connected by the approximate expression:

$$h = 17500 \operatorname{Log} \left[\frac{\delta_o}{\delta} \cdot \frac{T_o{}^{2/3}}{T} \cdot \frac{1}{f(V)^{2/3}} \right] \qquad (2)$$

Under these conditions, it is easy to show that IF AN AIRPLANE IS FLYING AT A GIVEN INCIDENCE THE DIFFERENCE BETWEEN THE ALTITUDE OF FLIGHT AND THE CEILING WILL ALWAYS BE THE SAME WHATEVER BE THE ALTITUDE OF FLIGHT.

In point of fact, by eliminating the speed from the two fundamental equations of horizontal flight of an airplane:

$$Q = \delta / \delta_o \cdot R_Y \cdot V^2$$

$$P_m \; \rho_o \cdot T/T_o \cdot f(V) = R_X \cdot V^3$$

we obtain the relation

$$\frac{P_m{}^{2/3} \cdot \rho_o{}^{2/3}}{Q} \quad \frac{(\delta_o / \delta)}{f(V)^{2/3}} \; : \; \frac{(T_o/T)^{2/3}}{(R_Y/R_X{}^{2/3})} \qquad (3)$$

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* I have established the values of this function in the last publication of the works of the Eiffel Laboratory: "Resumé of the principal works executed during the War ...," p. 100.

Let h_1, H_1, and h_2, H_2, be two groups of values of the altitude of flight and the height of the ceiling, corresponding to different powers, but such that the flights at altitudes h_1 and h_2, as well as at the ceilings H_1 and H_2, take place in each case at the same incidence.

Under these conditions we have:

$$17500 \text{ Log} \left[\frac{\delta_{H_2} \cdot T_{H_2}^{2/3} f(V_{h_2})^{2/3}}{\delta_{h_2} \cdot T_{h_2}^{2/3} f(V_{H_2})^{2/3}} \right] = 17500 \text{ Log} \left[\frac{\delta_{H_1} \cdot T_{H_1}^{2/3} \cdot f(V_{h_1})^{2/3}}{\delta_{h_1} \cdot T_{h_1}^{2/3} f(V_{H_1})^{2/3}} \right]$$

or

$$H_2 - h_2 = H_1 - h_1 \qquad \ldots$$

We stated above, p. 5, that ~~THE~~ SPEED RATIOS COULD BE INCREASED PROPORTIONALLY TO $\sqrt{\delta 2000 / \delta H\text{-}2000}$ BY INCREASING THE ALTITUDE OF THE CEILING WHILE KEEPING THE SAME ALTITUDE OF FLIGHT BELOW THE CEILING.

In fact, as we have just seen, the airplane will fly in both cases at the same incidence, that is, at the same value of R_Y and as

$$V = \sqrt{(Q/R_Y \cdot \delta / \delta_o)}$$

the maximum speed at altitude (H - 2000 m) will be increased in the ratio of $\sqrt{\delta \cdot 2000 / \delta H\text{-}2000}$, the minimum speed near the ground remaining the same.

IF THE INCREASE OF SPEED IS OBTAINED BY INCREASING BOTH THE ALTITUDE OF THE CEILING AND THE HEIGHT SEPARATING THE CEILING FROM THE ALTITUDE OF FLIGHT AT MAXIMUM SPEED, THE INCREASE IN THE RATIO WILL BE:

$$\sqrt{\frac{\delta_{4000}}{\delta_H}} \cdot 1.08 \sqrt[3]{\frac{T_o}{T_{H-2000} f(V_{H-2000})}}$$

As a matter of fact, according to relation (1) we may write that the increase of speed at altitude H-2000 m is:

$$\sqrt{\frac{\delta_{2000}}{\delta_{H-2000}}} = \sqrt{\frac{\delta_{4000}}{\delta_H}}$$

There is a second increase of 8% due to flight at smaller drag; finally, the third increase is proportional to the cube root of the increase of useful power due to the increase of temperature, of efficiency, and of the number of revolutions. As regards the value of $f(V_{H-2000})$ however, it may be equal to unity if the propeller has been specially calculated for flight at the altitude H-2000.

Se ction 2. - VALUES OF THE MOTIVE POWER IN FUNCTION OF THE ALTITUDE OF FLIGHT AT MAXIMUM SPEED.

1st Case. - ORDINARY ENGINE.

Equation (3) may be written as follows:

$$P_m = Q^{3/2} \cdot \frac{1}{\rho_o} \quad \frac{1}{R_Y^{3/2}/R_X} \quad \frac{(\delta_o/\delta)^{3/2} \cdot (T_o/T)}{f(V)} \tag{4}$$

Let P'_m, P''_m be the motive powers of the same airplane flying at the same incidence at altitudes where the specific weights of the air are respectively δ' and δ'' and the absolute temperatures T' and T".

Conseq uently, by equation (4) we have

$$P''_m = P'_m \cdot (\delta'/\delta'')^{3/2} \cdot (T'/T'') \cdot f(V'')/f(V') \cdot \rho'_o/\rho''_o$$

By equation (2) we may write, admitting $\rho'_o = \rho''_o$:

$$\text{Log}\ \frac{P''_m}{P'_m} = \frac{3}{2}\ \frac{H''-H'}{1750} = \frac{H''-H'}{11650}$$

where H' and H" are the respective ceilings of the airplanes in the two cases.

2nd Case. - SUPERCHARGED ENGINE.

In the case of a supercharged engine geared to a variable pitch propeller and keeping constant power P''_m at all altitudes, we have:

$$P_m'' = Q^{3/2} \cdot \frac{1}{\rho''_o} \cdot \frac{1}{R_Y^{3/2}/R_X} \cdot (\delta_o/\delta'')^{1/2} \tag{5}$$

Dividing expression (5) by expression (4) we obtain:

$$P_m'' = P_m' \cdot \frac{(\delta_o/\delta'')^{1/2}}{(\delta_o/\delta')^{3/2}\ (T_o/T')} \cdot \frac{\rho'_o\ f(V')}{\rho''_o}$$

If we assume $\rho'_o = \rho''_o$ we have:

$$P_m''/P_m' = \sqrt{\delta_o/\delta''} \cdot \left[(\delta'/\delta_o)^{3/2} \cdot T'/T_o \cdot f(V')\right]$$

Section 3. - VALUES OF THE SPEED RATIOS IN FUNCTION OF THE CHARACTERISTIC MECHANICAL AND AERODYNAMICAL COEFFICIENTS OF THE AIRPLANE.*

Combining the equation of flight near the ground at minimum speed (V_m)

$$Q = K_Y^m \, . \, S \, . \, V_m^2$$

with the equation of flight at altitude (δ) at maximum speed (V)

$$P_m \, \rho_0 \, f(V) \, . \, T/T_o \, . \, \sigma/\sigma_o = Q \, . \, K_X/K_Y \, . \, V \qquad (6)$$

we have:

$$E = V/V_m = \sqrt{S/Q} \, . \, P_m/Q \times T/T_o \, . \, \sigma/\sigma_o \, . \, \rho_o \, f(V) \times \frac{\sqrt{K_Y^m}}{K_X/K_Y} \qquad (7)$$

which gives the value of the speed ratio in function of the load per sq.m. and per H.P., and also the characteristic coefficients of the reduced polar of the airplane.

By means of expression (6) written for the ceiling, we eliminate the value of $\sqrt{S/Q} \, . \, P_m/Q$ in equation (7) and obtain:

$$E = \frac{\left(\frac{\sigma_o}{\delta_H}\right)^{3/2} \, . \, \frac{T_o}{T_H}}{\frac{\sigma_o}{\sigma} \, . \, \frac{T_o}{T}} \times \frac{f(V)}{f(V_H)} \times \frac{\sqrt{K_Y^m}}{\frac{K_X}{K_Y} \, . \left(\frac{K_Y^{3/2}}{K_X}\right)_H}$$

By this expression we can determine the speed ratios, when we take the ceiling, $\left[(\sigma_o/\delta_H)^{3/2} \, . \, T_o/T_H \, . \, 1/f(V_H)\right]$, the altitude of utilization $\left[\sigma_o/\sigma \, . \, T_o/T \, . \, 1/f(V)\right]$, and the aerodynamic coefficients characterizing the ceiling $\left[(K_Y^{3/2}/K_X)H_m\right]$, the altitude of utilization (K_X/K_Y) and the flight near the ground (K_Y^m).

This latter formula shows clearly that, as we have said on p.7, for a given type of airplane (**) the speed ratio depends solely on the ceiling and the altitude of utilization.

EXAMPLE: For an airplane of the 1918 type for which H = 4000 m and the altitude of utilization is 2000 m, we have:

$$E = \frac{1.51^{3/2} \times 1.09 \times \frac{1}{0.960}}{1.22 \times 1.02 \times \frac{1}{0.980}} \times \frac{0.063}{0.13 \times 1.68} = 1.62 \times 1.16 = 1.88$$

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* See p. 16.

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* By characteristic mechanical coefficients of the airplane, I mean the ratios Q/S and Q/P_m, which are the load per sq.m. and the load per H.P.; the characteristic aerodynamical coefficients are the values of K_Y^m maximum lift; $K_Y^{3/2}/K_X$ max., K_X/K_Y at altitude of practical utilization and K_X min., relative to its reduced polar.

** Airplanes of the same type have the same reduced polar.